Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Monday 31 October 2022 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

9 pages

8822–7104

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

$y = mx + b$

$\boxed{0 = b}$

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

$y = mx + b$

1. [Maximum mark: 7] $-2x + 3 = 0$

$(0,0)$ Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$. $y = 3$ $x = \frac{3}{2}$ $3 = -2 \cdot \frac{3}{2} + 3$

(a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. $\to$ Same slope [2]

(b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$. $\frac{-1}{-2} = \frac{1}{2}m$ [3]

(c) Find $(g \circ h)(0)$. [2]

(b) perpendicular so the slope $= m = \frac{-1}{-2} = \frac{1}{2}$

$y = mx + b \Rightarrow 2 = -1 \cdot \frac{1}{2} + b \Rightarrow 2 = -\frac{1}{2} + b$

$b = 2 + \frac{1}{2} = \frac{5}{2}$

$h(x) = \frac{1}{2}x + \frac{5}{2}$

✓3

(c) $= g(h(0)) = \frac{1}{2} \cdot 0 + \frac{5}{2}$

$g\left(\frac{5}{2}\right) \to -2 \cdot \frac{5}{2} = -5$

✓2

(a) $g(x) = -2x$

✓2

2. [Maximum mark: 4]

The function g is defined by $g(x) = e^{x^2+1}$, where $x \in \mathbb{R}$.

Find $g'(-1)$.

$g(x) = e^{x^2+1}$ $g'(x) = 2xe^{x^2+1}$

$g'(-1) = -2e^2$ //

✔ 4

3. [Maximum mark: 7]

Consider a circle with a diameter AB, where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

(a) Find

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. [4]

(1, 4, 0)

-3, 2, -4

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale

M1

$-3, 2, -4$

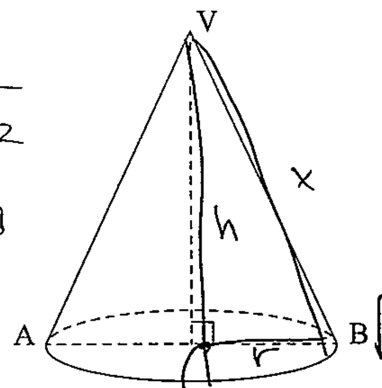
$+\ 1, 4, 0$

A1

$-2, 6, -4 / 2$

$-1, 3, -2$

~~cemme~~ radius



$h^2 + r^2 = x^2$

$\sqrt{h + r} = x$

$(-1, -1, 0)$

(b) Find the exact volume of the cone. [3]

$\frac{1}{3}\pi (r^2) \cdot h$

A0

4. [Maximum mark: 5]

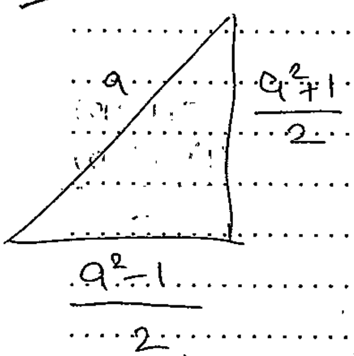
Let a be a constant, where $a > 1$.

(a) Show that $a^2 + \left(\frac{a^2-1}{2}\right)^2 = \left(\frac{a^2+1}{2}\right)^2$. [3]

Consider a right-angled triangle with sides of length a, $\left(\frac{a^2-1}{2}\right)$ and $\left(\frac{a^2+1}{2}\right)$.

(b) Find an expression for the area of the triangle in terms of a. [2]

b.



Area

$\frac{1}{2}\left(\frac{a^2-1}{2}\right)\left(\frac{a^2+1}{2 \cdot 4}\right) \cdot \text{Sinc}$

a

$\frac{(a^2-1)^2}{(2)^2} = \frac{a^4+1}{4} + a^2$

rearrange

$\frac{4a^2}{4} + \frac{a^4+1}{4} = \frac{4a^2+a^4+1}{4}$

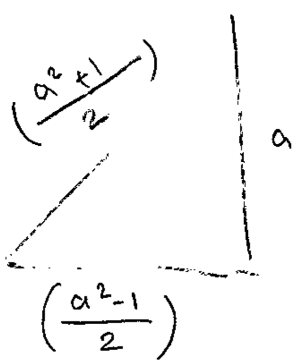
$\frac{\sqrt{a^4+4a^2+1}}{\sqrt{4}} = \frac{a^2+2a+1}{2}$

A0

$\frac{a^2+2a+1}{2} \rightarrow \frac{(a^2+1)^2}{2}$ //

$(a^2-1)(a^2+1) = a^2-1^2$

$\frac{a^2-1^2}{4 \cdot 2 = 8} \cdot \text{Sinc} = \frac{a^2-1^2}{8} \cdot \text{sinc}$ //



$\frac{1}{2} \cdot a \cdot \left(\frac{a^2-1}{2}\right) \cdot \text{sinc}$

$\frac{a}{2} \cdot \frac{a^2-1}{2} = \frac{a^3-a}{4} \cdot \text{sinc}$

A1

A0

$x^2+1 \Rightarrow \frac{x^3}{3}+x$

5. [Maximum mark: 5]

The derivative of the function f is given by $f'(x)=\frac{6x}{x^2+1}$.

The graph of $y=f(x)$ passes through the point $(1, 5)$. Find an expression for $f(x)$.

$f(x)=\int f'(x)\,dx$ M1

$\int \frac{6x}{x^2+1}\,dx \qquad u=x^2+1 \qquad \frac{du}{dx}=2x$

$dx=\frac{du}{2x}$

~~raise the power by one and divide it~~

A1 $\int \frac{\cancel{6x}}{u}\cdot\frac{du}{\cancel{2x}}=\int\frac{3}{u}\,du \rightarrow \int 3\cdot u^{-1}\,du$

$\cancel{u^{-1+1}=u^0}$

$\cancel{3\int u^{-1}\,du = 3}$

$3\int\frac{1}{u}\,du$

A0 $3\int\frac{1}{u^2}+c$

$f(x)=\frac{3}{u^2}+c=\frac{3}{x^4+1}+c$

$5-\frac{3}{2}$

$\frac{10-3}{2}=\frac{7}{2}$

M1 $f(1)=5 \qquad f(1)=\frac{3}{2}+c=5$

$c=\frac{7}{2}$

A0

$f(x)=\frac{3}{x^4+1}+\frac{7}{2}$ //

6. [Maximum mark: 6]

(0.3) (0.8)

0.1

Events A and B are such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$.

(a) Determine the value of $\mathrm{P}(A \cap B)$ in the case where the events A and B are independent. [1]

(b) Determine the minimum possible value of $\mathrm{P}(A \cap B)$. → 0.1 0.3 0.8 [3]

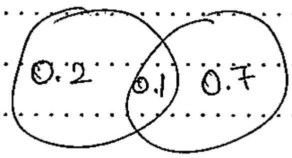
(c) Determine the maximum possible value of $\mathrm{P}(A \cap B)$, justifying your answer. → 0.3 [2]

(a) Since they're independent ⟹ $P(A \cap B) = P(A)P(B)$

$P(A \cap B) = 0.3 \times 0.8 = 2.4$

A0

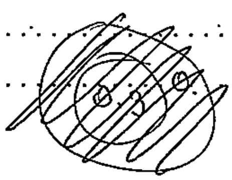
(b) minimum possible value is 0.1



$= 0.2 + 0.1 + 0.7 = 1$

✓ 3

(c) ~~Maximum value is 0.3~~



The maximum possible value is ~~[illegible]~~ 0.1 because if the intersection is greater than 0.1, the possibility is not 1

A0

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

(a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

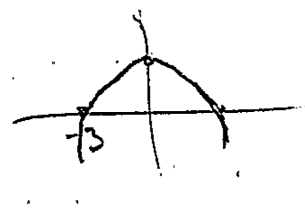
$a = 5$ $h = 3$ $k = 2$

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

(b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t;

(ii) find the range of g. [7]



(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

8. [Maximum mark: 15]

$ab^a = 3$

(a) Calculate the value of each of the following logarithms:

(i) $\log_2 \frac{1}{16}$; $\log_2 1/16 = 2^x = 16^{-1}$

-4 (ii) $\log_9 3$; $9^x = 3$ $2^x = 2^{-4}$ $2^x =$

(iii) $\log_{\sqrt{3}} 81$. $3^{2x} =$ $x = -4$ $x = \frac{1}{2}$ [7]

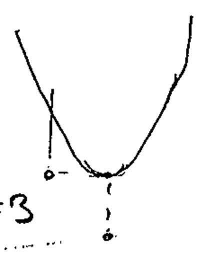
$\frac{1}{2}$ (b) It is given that $\boxed{\log_{ab} a = 3}$, where $a, b \in \mathbb{R}^+$, $ab \neq 1$.

(i) Show that $\boxed{\log_{ab} b = -2}$. $\log_{ab} a = 3$ $ab^a = 3$

8

(ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. $\sqrt{3}^x = 81$ [8]

$3^{1/2x} = 3^4$



$\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3}$

3 3 $\frac{1}{2} 3 = 4$ 3

$\log_{ab} a = 3$

9 $x = \frac{2 \cdot 2}{\frac{1}{2}} \cdot 2^3$

$x = 8$

Do **not** write solutions on this page.

9. [Maximum mark: 15]

The function f is defined by $f(x) = \cos^2 x - 3\sin^2 x,\ 0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = f(x)$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [3]

$f(x) = \cos^2 x - 3\sin^2 x$

$3(-1\cos$

3

$\cos^2 x - 3\sin^2 x = 0$

$\sin^2 + \cos^2 = 1$

$\boxed{1 - \sin^2 x - 3\sin^2 x = 0}$

$1 - \cos^2$

$-\sin^2 x - 3\sin^2 x - 1 = 0$

$-4\sin^2 x - 1 = 0$

SEEN

ANSWER BOOKLET
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7 (a) $x=5$ $h=3$ $k=2$

(i)

(b) $g(-3) = p(-3)^2 + (t-1) - 3 - p = 4$

$9p^2 + 3t + 3 - 4 - p = 0$

$9p^2 - \cancel{} - p + 3t - 1 = 0$

$g(1) = p(1)^2 + (t-1)(1) - p = 4$

$p^2 + t - 1 - p - 4 = 0$

$p^2 - p - 5 + t = 0$

factoring them

(ii) The range is ~~~~ $-3 \leq x \leq 4$

Solutions continue...

please turn the page.

8

(a)(i) $\log_2 \frac{1}{16} = \quad 2^x = 16^{-1} \rightarrow 2^x = 2^{-4} \quad x = -4$

$\log_2 \frac{1}{16} = -4$ //

(ii) $\log_9 3 = \quad 9^x = 3 = 3^{2x} = 3^1 \quad 2x = 1 \quad x = \frac{1}{2}$

$\log_9 3 = \frac{1}{2}$

(iii) $\log_{\sqrt{3}} 81 = \quad 3^{1/2x} = 3^4 \quad \frac{1}{2}x = 4 \quad x = 8$ //

$\log_{\sqrt{3}} 81 = 8$ //

✓ 7

(b) $\log_{ab} a = 3$

$ab^3 = a$

$a^3 = ab$

A0

$b = a^2$ //

$\log_{ab} a^2 =$

$ab^x = a^2$

A0

$ab^{2x} = a$

$c^2 - 3s^2$

9

(a) $f(x) = \cos^2 x - 3\sin^2 x$

~~$\cos^2 x - 3\sin^2 x = 0$~~

~~$(1 - \sin^2 x) - 3\sin^2 x = 0$~~

~~$\sin^2 x + 3\sin^2 x - 1 = 0$~~

$\cos^2 x - \underbrace{3\sin^2 x}_{1 - \cos^2 x} = 0$

$\cos^2 x - 3(1 - \cos^2 x) = 0$ M1

$\cos^2 x - 3 - 3\cos^2 x = 0.$ 0

$\cos^2 x - 3\cos^2 x - 3 = 0$

$-3\cos^2 x + \cos^2 x - 3 = 0$

(b) $f'(x) = \overset{\text{chain rule}}{2} \cdot -\sin x - 6\cos x$

$= -2\sin x - 6\cos x$ // A0

04AX03